MUTUAL COUPLING BETWEEN PARALLEL PLATE WAVEGUIDES

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Abstract

Coupling between parallel plate waveguides is investigated using the Wiener-Hopf technique. For a $TE_{o,\ell}$ excitation of one guide, exact solutions for the radiated, reflected and transmitted fields are found and are expressed by three terms. One term is due to the exciting waveguide alone and the other two are due to interaction between two waveguides. The expressions for the exact solutions are further reduced to derive approximate solutions, which could be obtained using the ray theory of diffraction with a modified diffraction coefficient.

Introduction

Open ended waveguide structures have recently received considerable attention as radiating elements[1,2]. Mutual coupling between such structures has practical significance and has been studied by various methods. The equivalent static approach[3] and the ray theory of diffraction[4] have provided solutions for many interesting problems but have limitations due to difficulties in including higher order rays[5,6].

An alternate approach based on Wiener-Hopf technique[7] provides solutions for certain classes of boundary value problems. In this paper, the boundary value problem concerning two parallel plate waveguides having the same width and the same axis of symmetry is formulated and is solved using Wiener-Hopf technique. The final results are expressed in terms of an integral extending from zero to infinity, which is then evaluated numerically[8] using the Gauss-Laguerre quadrature formula[9].

The solutions are also reduced to those obtained using the ray theory of diffraction. The transformed Green's function $G(\alpha)$ associated with the Wiener-Hopf equation is expanded in a power series, which after integration and expansion of each term, the ray theory solutions are found by retaining the first term only. It is found that the resulting series are convergent if $[(Kd)^2/4KL]<1$, where d and L are the width and separation distance of two waveguides. The ray theory solutions are found by using the first term together with the modified diffraction coefficient of Lee[10,11].

The exact and approximate solutions are divided into three terms. The first term gives the contribution due to exciting waveguide only, where as the second and third terms give the contribution due to coupling between two waveguides.

Formulation of the Problem

Consider two infinitely thin and perfectly conducting parallel plate waveguides of width $2a$ and separated by a distance L and located in free space as shown in Figure 1. With a time factor $e^{-i\omega t}$, an incident $TE_{o,\ell}$ mode, ℓ odd, is assumed to be propogating in the exciting waveguide along the positive z-direction, in the form

$$E_y{}^i = \phi^i(x,z) = \cos(\frac{\ell\pi}{2a}x)e^{-\gamma_\ell z} \qquad (1)$$

where $\gamma_\ell = [(\ell\pi/2a)^2 - K^2]^{\frac{1}{2}}$ and $K = K_1 + i K_2$ is the propagation constant in free space. Using Jones's method of formulating[12] the following modified Wiener-Hopf equation of second kind is obtained

$$J_-(\alpha) + e^{i\alpha L} J_+(\alpha) + \Phi_1(a,\alpha)/G(\alpha) = \frac{i\pi\ell}{2a\sqrt{2\pi}}(-1)^{\frac{\ell-1}{2}} \frac{1-e^{(i\alpha - \gamma_\ell)L}}{\alpha + i\gamma_\ell}$$

$$|\tau| < K_2 \qquad (2)$$

where $\alpha = \sigma + i\tau$, $G(\alpha) = \cosh \gamma a/\gamma \exp(\gamma a)$ and $\Phi_1(a,\alpha)$ is the Fourier transform of aperture electric field to be determined. $J_+(\alpha)$ and $J_-(\alpha)$ are unknown and are analytic in the upper $(\tau > -K_2)$ and lower $(\tau < K_2)$ halves of the α-plane, respectively. It can be shown that $\Phi_1(a,\alpha)$ satisfies

$$\Phi_1(a,\alpha) = \frac{1}{2} G(\alpha)[e^{i\alpha L}\{S(-\alpha)-D(-\alpha)\}-\{S(\alpha)+D(\alpha)\}] \qquad (3)$$

where

$$\begin{matrix} S(\alpha) \\ D(\alpha) \end{matrix} = J_-(\alpha) \mp J_+(-\alpha) - \frac{i\pi\ell}{2a\sqrt{2\pi}}(-1)^{\frac{\ell-1}{2}}[\frac{1}{\alpha+i\gamma_\ell} \mp \frac{e^{-\gamma_\ell L}}{\alpha-i\gamma_\ell}] \qquad (4)$$

The upper and lower signs belong respectively to $S(\alpha)$ and $D(\alpha)$. These functions satisfy the following integral equation

$$\frac{i\pi\ell}{2a\sqrt{2\pi}}(-1)^{\frac{\ell-1}{2}} \frac{G_+(i\gamma_\ell)}{\alpha+i\gamma_\ell} + G_-(\alpha)E(\alpha) = \int_{-\infty-id}^{\infty-id} \frac{G_+(\beta)E(\beta)e^{-i\beta L}}{(\beta+\alpha)(2\pi i/\lambda)} d\beta$$

$$-K_2 < -d < \tau < d < K_2 \qquad (5)$$

where

$$E(\alpha) = \begin{cases} S(\alpha), & \lambda = 1 \\ D(\alpha), & \lambda = -1 \end{cases}$$

and $G_+(\alpha)$ is the "plus part" of $G(\alpha)$ $(G(\alpha) = G_+(\alpha)G_-(\alpha))$. A solution of the above integral equation together with equation (3) gives $\Phi_1(a,\alpha)$ and hence $\Phi(x,\alpha)$ can be determined. The final solution of the scaler potential $\phi(x,z)$ can be found by an inverse Fourier transform. To determine $E(\alpha)$ one notes that the right handside of equation (5) is of the form

$$I = \int_{-\infty-id}^{\infty-id} \frac{G_+(\beta)E(\beta)}{\beta+\alpha} e^{-i\beta L} d\beta = \int_{-\infty-id}^{\infty-id} \frac{\alpha \cosh\gamma a \cdot e^{-\gamma a} E(\beta) e^{-i\beta L}}{\gamma a \cdot (\beta+\alpha)\, G_-(\beta)} d\beta \tag{6}$$

For large L, the major contribution is from the integral over a small neighborhood around the branch point $\beta = -K$. Deforming the contour in the lower half plane and expanding $G_-(\beta)$ and $E(\beta)$ in a taylor series about the branch point $\beta = -K$, the first term gives

$$I = a \frac{E(-K)}{G_-(-K)} \; T(\alpha) \tag{7}$$

where

$$T(\alpha) = 2 \int_{-K}^{-K-i\infty} \frac{\cosh^2\gamma a}{\gamma a(\beta+\alpha)} e^{-i\beta L} d\beta =$$

$$\frac{2L}{a} \int_0^{\infty} \frac{\cosh^2[\frac{a}{L}\sqrt{2i\, KL\, U - U^2}]e^{-U}}{\sqrt{2i\, KL\, U - U^2}\,[U + i\, KL(\frac{\alpha}{K} - 1)]} dU \tag{8}$$

The new variable U in the above integral is chosen to transfer the integral to a form suitable for numerical integration. Equation (7) with (5) and (3) gives

$$\Phi_1(a,\alpha) = \frac{i\pi\ell}{2a\sqrt{2\pi}}(-1)^{\frac{\ell-1}{2}} G_+(i\gamma_\ell)\{\frac{G_+(\alpha)}{\alpha+i\gamma_\ell} - \frac{a/2\pi i}{(i\gamma_\ell - K)(1-F^2)G_+^2(K)}$$

$$[F\; T(\alpha)\; G_+(\alpha) + T(-\alpha)\; G_+(-\alpha)\; e^{i\alpha L}]\} \tag{9}$$

where

$$F = \frac{-a}{2\pi i} \; \frac{T(-K)}{G_+^{\,2}(K)} \tag{10}$$

I-Radiation field

Outside waveguides, the scattered electric field is given by

$$\phi^S(x,z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty+i\tau}^{\infty+i\tau} \Phi_1(a,\alpha) e^{\gamma(a-x)-i\alpha z} d\alpha \quad , \; |\tau|<K_2 \tag{11}$$

which with a saddle point integration at far zone gives

$$\phi^S(\rho,\theta) = \frac{e^{i(K\rho - \frac{\pi}{4})}}{\sqrt{K\rho}} K \sin\theta \; \Phi_1(a,\; K\cos\theta) e^{-i\; Ka\sin\theta} \tag{12}$$

Replacing for $\Phi_1(a,\alpha)$ from (a), the radiated field $\phi S(\rho,\theta)$ is given by three terms. The first term is the field radiated from open end of the exciting waveguide alone, and the second and third terms are the contributions of multiple interactions between waveguides.

II-Reflected and transmitted fields

Inside the exciting waveguide the reflected electric field is given by

$$\phi_r(x,z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty+i\tau}^{\infty+i\tau} \Phi_1(a,\alpha) \frac{\cosh\gamma x}{\cosh\gamma a} e^{-i\alpha z} d\alpha \; , \; |\tau|<K_2 \tag{13}$$

After closing the contour in the upper half plane and using residue theorem one finds,

$$\phi_r(x,z) = \sum_{m=1,3,5}^{\infty} [(R_{\ell,m} + R^{(1)}_{\ell,m}) e^{\gamma_m z} + R^{(2)}_{\ell,m}(z)] \cos(\frac{m\pi}{2a}x) \tag{14}$$

Similarly inside the coupled waveguide the transmitted electric field is given by equation (13), but to evaluate the integral the contour must be closed in the lower half plane to give

$$\phi_t(x,z) = \sum_{m=1,3,5}^{\infty} [T_{\ell,m}(z) + T^{(1)}_{\ell,m}(z) + T^{(2)}_{\ell,m} e^{-\gamma_m z}] \cos(\frac{m\pi}{2\alpha}x) \tag{15}$$

Again the first terms in (14) and (15) are due to the exciting waveguide only, while other two terms are the contributions due to interaction (coupling) between two waveguides. $R^{(2)}_{\ell,m}(z)$, $T_{\ell,m}(z)$ and $T^{(1)}_{\ell,m}(z)$ are due to fields scattered by each waveguide alone and decay with z according to Sommerfeld radiation condition. Thus, at far distances from the openings the only contribution for the reflected and transmitted fields is due to the coefficients $R_{\ell,m}$, $R^{(1)}_{\ell,m}$ and $T^{(2)}_{\ell,m}$ respectively. The reflection and transmission coefficients which are functions of z can be evaluated numerically similar to $T(\alpha)$ in equation (8).

Reduction of the solution to that of ray theory of Diffraction

If the Green's function $G(\alpha)$ is expanded in a power series $T(\alpha)$ can be written as

$$T(\alpha) = \sum_{n=0,1,2,..}^{\infty} T_n(\alpha) \tag{16}$$

where

$$T_n(\alpha) = \frac{1}{\varepsilon_n} \int_{-\infty-id}^{\infty-id} \frac{(-2\gamma a)^n}{\gamma a \cdot n!} \frac{e^{-i\beta L}}{\beta+\alpha} d\beta, \quad \varepsilon_n = \begin{cases} 1 & ,\text{for } n = 0 \\ 2 & ,\text{for } n \neq 0 \end{cases} \tag{17}$$

In the neighborhood of $\beta = -K$, the function $(\beta-K)^{n-1/2}$ is regular and smooth and can be replaced by $(-2K)^{n-1/2}$. Therefore, deforming the contour in (17) it may be shown that, the final result is a Whittaker function, which after an expansion and retaining first term only gives

$$T(\alpha) = \frac{-2\pi i\; e^{i(KL - \frac{\pi}{4})}}{a\sqrt{2\pi\; KL}} \; \frac{1}{K-\alpha} \; [1 + i\nu - \nu^2 - \frac{2}{3} i\nu^3 + \ldots]$$

$$\nu = \frac{(Ka)^2}{KL} \tag{18}$$

Retaining only the first term in (18), its substitution in exact solutions of radiated, reflected and transmitted fields gives the results obtained by using ray theory of diffraction in conjunction with the modified diffraction coefficient of Lee. However, higher terms of $T(\alpha)$ provide corrections when L is not large enough.

Results and Discussions

Some computed results for the radiated power and reflected and transmitted fields for a waveguide size $2a/\lambda = 0.6$ and $TE_{0,1}$ excitation are shown in figures (2) to (4). The infinite integrals in the respective equations are computed by a Gauss-Laguerre quadrature formula with 15 intervals. As expected with decreasing KL, the direction of the main lobe of radiation moves progressively away from the forward direction. The reflected and transmitted fields are oscillating functions of KL with period π and decay gradually to reach their final values for $KL = \infty$.

Equation (18) shows that the series is convergent for $\nu < 1$, L large, and the solutions of diffraction theory can be obtained by retaining first term only. Higher order terms give correction terms when the separation distance L is not large enough.

Similar results for other excitation modes can be found by using proper Green's functions. The method may also be extended to study the coupling between parallel plate waveguides of different widths or arrays of waveguides.

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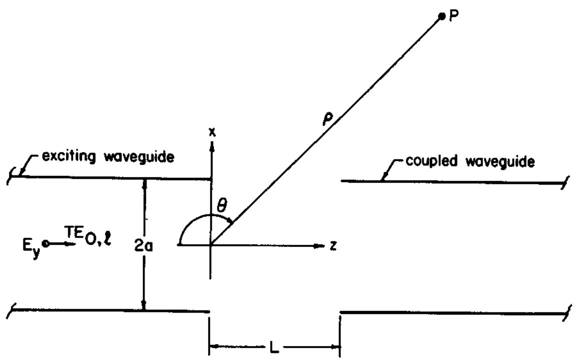


Fig. 1 Geometry of the problem

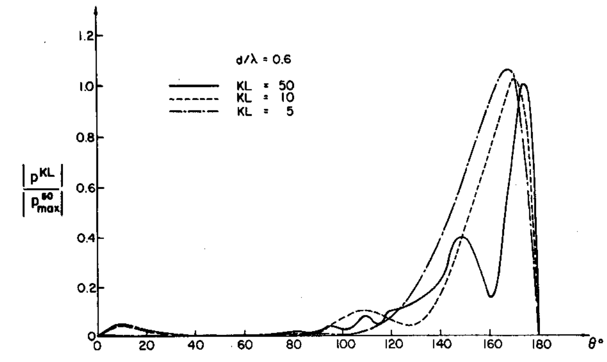


Fig. 2 Radiation Pattern of the $TE_{0,1}$ mode

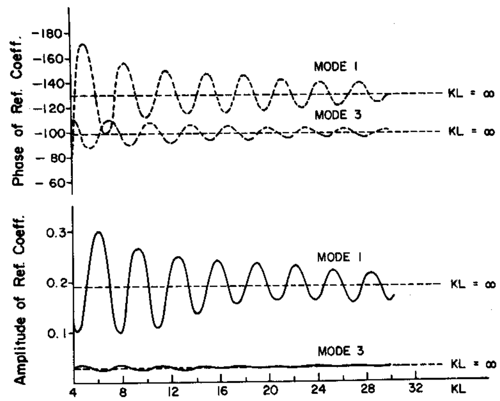
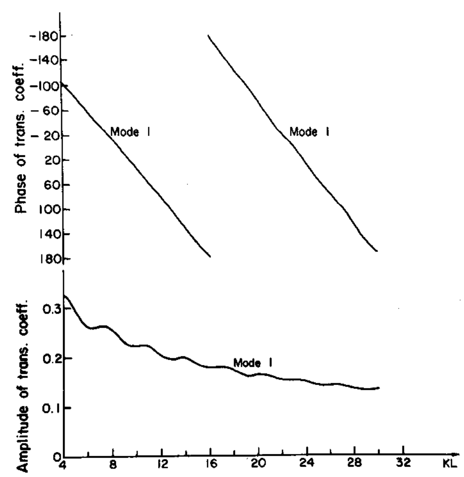


Fig. 3 Reflection Coefficients of the $TE_{0,1}$ mode for $d/\lambda = 0.6$



Trans. Coefficient of the $TE_{0,1}$ mode for $d/\lambda = 0.6$

Fig. 4